δ-DERIVATIONS OF CLASSICAL LIE SUPERALGEBRAS

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Abstract:

We consider the δ-derivations of classical Lie superalgebras and prove that these superalgebras admit nonzero δ-derivations only when $\delta = 0, \frac{1}{2}, 1$. The structure of $\frac{1}{2}$-derivations for classical Lie superalgebras is completely determined.

INTRODUCTION

Hopkins [1] considered the antiderivations of Lie algebras. An antiderivation is a special case of a δ-derivation — that is, a linear mapping μ of an algebra such that $\mu(xy) = \delta(\mu(x)y + x\mu(y))$, where δ is some fixed element of the ground field. Independently, these results were obtained in a generalized form by Filippov [2]. He proved that a prime Lie Φ-algebra equipped with a nondegenerate symmetric invariant bilinear form does not admit nonzero δ-derivations if $\delta \neq -1, 0, \frac{1}{2}, 1$. In the same article, a description of $\frac{1}{2}$-derivations was given for an arbitrary prime Lie Φ-algebra A ($\frac{1}{6} \in \Phi$) equipped with a nondegenerate symmetric invariant bilinear form. He proved that a linear mapping $\phi : A \to A$ is a $\frac{1}{2}$-derivation if and only if $\phi \in \Gamma(A)$, where $\Gamma(A)$ is the centroid of A. Let A be a simple Lie algebra over a field of characteristic $p \neq 2, 3$. Assume that A is equipped with a nondegenerate symmetric invariant bilinear form. It follows from the results cited above that every $\frac{1}{2}$-derivation ϕ of A is of the shape $\phi(x) = \alpha x$, $\alpha \in \Phi$. At a later time, Filippov described the δ-derivations for prime alternative and non-Lie Malcev Φ-algebras under some restrictions on the ring of operators Φ. He proved that the algebras in these classes do not admit nonzero δ-derivations if $\delta \neq 0, \frac{1}{2}, 1$ [3].

In [4], the δ-derivations were studied for simple finite-dimensional Jordan superalgebras over an algebraically closed field of characteristic 0 and for semisimple finite-dimensional Jordan algebras over an algebraically closed field of characteristic distinct from 2. It was proved that these classes of algebras and superalgebras possess nonzero δ-derivations only when $\delta = 0, \frac{1}{2}, 1$. A complete description of $\frac{1}{2}$-derivations was given for these classes, and it was shown that ϕ is a $\frac{1}{2}$-derivation if and only if $\phi(x) = \alpha x$ for some $\alpha \in F$.

In the present work, we give a description of nontrivial δ-derivations of classical Lie superalgebras. We prove that the superalgebras in this class admit nonzero δ-derivations only if $\delta = 0, \frac{1}{2}, 1$. We give a complete description of $\frac{1}{2}$-derivations for this class of superalgebras. We prove that ϕ is a $\frac{1}{2}$-derivation if and only if $\phi(x) = \alpha x$ for some $\alpha \in F$.

§ 1. Basic Facts and Definitions

We denote by $\mathrm{span}\langle a, b\rangle$ the linear span of elements a and b. Let U be a vector space, and let V be a subspace in U. Take $x \in U$ and denote by $x|_V$ the projection of x on V.

Let F be an algebraically closed field of characteristic 0. A Lie superalgebra $G = G_0 + G_1$ is a $\mathbb{Z}_2$-graded algebra with the superidentities

$$[x, y] = -(-1)^{p(x)p(y)}[y, x], \quad [[x, y], z] - [x, [y, z]] - (-1)^{p(x)p(z)}[[x, z], y] = 0, \tag{1}$$

where $p(x) = i$ if $x \in G_i$. The second superidentity is a generalization of the Jacobi identity.

A finite-dimensional Lie superalgebra $G = G_0 + G_1$ is called *classical* provided that G is simple and the representation of G_0 on G_1 is completely reducible. In [5], Kac gave a complete classification of classical Lie superalgebras over an algebraically closed field of characteristic 0.

Theorem 1 [5]. Let A be a classical Lie superalgebra over an algebraically closed field of characteristic 0. Then A is one of the superalgebras

$$A(m, n),\ A(n, n),\ B(m, n),\ D(m, n),\ C(n),\ P(n),\ Q(n),\ D(2, 1; \beta),\ G(3),\ F(4).$$

Now, we recall a description of the superalgebras mentioned in Theorem 1.

$A(m, n)$: Let $sl(m, n)$ be the following subspace in the matrix superalgebra $M_{m+n,m+n}$ with the $\mathbb{Z}_2$-grading:

$$\begin{aligned}
(sl(m, n))_0 &= \left\{ \begin{pmatrix} A & 0 \\ 0 & D \end{pmatrix} : A \in M_m(F), D \in M_n(F), tr(A) = tr(D) \right\}, \\
(sl(m, n))_1 &= \left\{ \begin{pmatrix} 0 & B \\ C & 0 \end{pmatrix} : B \in M_{m,n}(F), C \in M_{n,m}(F) \right\}.
\end{aligned}$$

Equip $sl(m, n)$ with the structure of a Lie superalgebra by $[a, b] = ab - (-1)^{p(a)p(b)}ba$. If $m = n$ then this superalgebra contains the one-dimensional ideal $\langle E_{2n}\rangle$ consisting of the scalar matrices $\lambda E_{2n}, \lambda \in F$. The Lie superalgebra $sl(1, 1)$ is three-dimensional and nilpotent. We set $A(m, n) = sl(m + 1, n + 1)$ for $m \neq n$, $m, n \geq 0$, and $A(n, n) = sl(n + 1, n + 1)/\langle E_{2n+2}\rangle$ for $n > 0$.

$B(m, n)$, $D(m, n)$, $C(n)$, $P(n)$, $Q(n)$ are some subsuperalgebras in $A(k, l)$.

$D(2, 1; \alpha)$, $\alpha \in F^*\backslash\{0, -1\}$: This is a one-parameter family of 17-dimensional Lie superalgebras consisting of all simple Lie superalgebras for which $(D(2, 1; \alpha))_0$ is a Lie algebra of type $G_1^1 \oplus G_1^2 \oplus G_1^3$ ($G_1^j \cong A_1$) and its representation on $(D(2, 1; \alpha))_1$ is $sl_2 \otimes sl_2 \otimes sl_2$.

$F(4)$: Define $F(4)$ as a 40-dimensional classical Lie superalgebra for which $(F(4))_0$ is a Lie algebra of type $B_3 \oplus A_1$ and its representation on $(F(4))_1$ is $\mathrm{spin}_7 \otimes sl_2$.

$G(3)$: Define $G(3)$ as a 31-dimensional classical Lie superalgebra for which $(G(3))_0$ is a Lie algebra of type $\mathbf{G}_2 \oplus A_1$ and its representation on $(G(3))_1$ is $G_2 \otimes sl_2$.

The classical superalgebras distinct from $Q(n)$, $P(n)$, and $A(1, 1)$ were called the *basic classical Lie superalgebras* in [5]. We will use this terminology.

Below we describe the systems of roots for the basic classical Lie superalgebras [5]. Denote by Δ_0 and Δ_1 the systems of even and odd roots respectively. Denote by Π a simple system of roots. In these cases, a Cartan subalgebra H is a subspace of the space D of diagonal matrices. The roots are expressed in terms of the standard basis ϵ_i on the dual space D^* (more accurately, the restrictions of ϵ_i on H).

$A(m,n)$: The root system is expressed in terms of linear functionals $\epsilon_1,\ldots,\epsilon_{m+1},\delta_1=\epsilon_{m+2},\ldots,\delta_{n+1}=\epsilon_{n+m+2}$:

$$\Delta_0=\{\epsilon_i-\epsilon_j;\delta_i-\delta_j\},\ i\neq j;\quad \Delta_1=\{\pm(\epsilon_i-\delta_j)\};$$

$$\Pi=\{\epsilon_1-\epsilon_2,\ldots,\epsilon_{m+1}-\delta_1,\delta_1-\delta_2,\ldots,\delta_n-\delta_{n+1}\}.$$

$B(m,n)$: The root system is expressed in terms of $\epsilon_1,\ldots,\epsilon_m,\delta_1=\epsilon_{2m+1},\ldots,\delta_n=\epsilon_{2m+n}$:

$$\Delta_0=\{\pm\epsilon_i\pm\epsilon_j;\pm2\delta_i;\pm\epsilon_i;\pm\delta_i\pm\ delta_j\},\ i\neq j;\quad \Delta_1=\{\pm\delta_i;\pm\epsilon_i\pm\delta_j\};$$

$$\Pi=\{\delta_1-\delta_2,\ldots,\delta_n-\epsilon_1,\epsilon_1-\epsilon_2,\ldots,\epsilon_{m-1}-\epsilon_m,\epsilon_m\},\ m>0,$$
$$\Pi=\{\delta_1-\delta_2,\ldots,\delta_{n-1}-\delta_n,\delta_n\},\ m=0.$$

$C(n)$: The root system is expressed in terms of $\epsilon=\epsilon_1,\delta_1=\epsilon_3,\ldots,\delta_{n-1}=\epsilon_{n+1}$:

$$\Delta_0=\{\pm2\delta_i;\pm\delta_i\pm\delta_j\};\quad \Delta_1=\{\pm\epsilon\pm\delta_i\};$$

$$\Pi=\{\delta_1-\delta_2,\ldots,\delta_{n-2}-\delta_{n-1},\delta_{n-1}-\epsilon,\delta_{n-1}+\epsilon\}.$$

$D(m,n)$: The root system is expressed in terms of $\epsilon_1,\ldots,\epsilon_m,\delta_1=\epsilon_{2m+1},\ldots,\delta_n=\epsilon_{2m+n}$:

$$\Delta_0=\{\pm\epsilon_i\pm\epsilon_j;\pm2\delta_i;\pm\delta_i\pm\delta_j\},\ i\neq j;\quad \Delta_1=\{\pm\epsilon_i\pm\delta_j\};$$

$$\Pi=\{\epsilon_1-\epsilon_2,\ldots,\epsilon_m-\delta_1,\delta_1-\delta_2,\ldots,\delta_{n-1}-\delta_n,2\delta_n\}.$$

$D(2,1;\alpha)$: $\Delta_0=\{\pm2\epsilon_1,\pm2\epsilon_2,\pm2\epsilon_3\}$; $\Delta_1=\{\pm\epsilon_1\pm\epsilon_2\pm\epsilon_3\}$; $\Pi=\{\epsilon_1+\epsilon_2+\epsilon_3,-2\epsilon_1,-2\epsilon_2\}$.

$F(4)$: The root system is expressed in terms of the functionals $\epsilon_1,\epsilon_2,\epsilon_3$, corresponding to B_3, and δ, corresponding to A_1:

$$\Delta_0=\{\pm\epsilon_i\pm\epsilon_j;\pm\epsilon_i;\pm\delta\},\ i\neq j;\quad \Delta_1=\{1/2(\pm\epsilon_1\pm\epsilon_2\pm\epsilon_3\pm\delta)\};$$

$$\Pi=\{1/2(\epsilon_1+\epsilon_2+\epsilon_3+\delta),-\epsilon_1,\epsilon_1-\epsilon_2,\epsilon_2-\epsilon_3\}.$$

$G(3)$: The root system is expressed in terms of $\epsilon_1,\epsilon_2,\epsilon_3$, corresponding to $\mathbf{G}_2$, $\epsilon_1+\epsilon_2+\epsilon_3=0$, and δ, corresponding to A_1:

$$\Delta_0=\{\epsilon_i-\epsilon_j;\pm\epsilon_i;\pm2\delta\};\quad \Delta_1=\{\pm\epsilon_i\pm\delta;\pm\delta\};\quad \Pi=\{\delta+\epsilon_1,\epsilon_2,\epsilon_3-\epsilon_2\}.$$

In what follows, G_β denotes the root subspace corresponding to the root β, and g_β denotes an element of this space.

For the basic classical Lie superalgebras, we have the following

Theorem 2 [5]. Let G be a basic classical Lie superalgebra, and let $G=\oplus G_\alpha$ be its root decomposition with respect to a Cartan subalgebra H. Then

(a) $G_0=H$,

(b) $\dim(G_\alpha)=1$ for $\alpha\neq0$,

(c) $[G_\alpha,G_\beta]\neq0$ if and only if $\alpha,\beta,\alpha+\beta\in\Delta$.

Take $\delta\in F$. A linear mapping ϕ of a superalgebra A is called a *δ-derivation* provided that for arbitrary $x,y\in A$ holds

$$\phi(xy)=\delta(x\phi(y)+\phi(x)y). \tag{2}$$

The definition of a 1-derivation coincides with the conventional definition of a derivation; a 0-*derivation* is an arbitrary endomorphism ϕ of A such that $\phi(A^2) = 0$. By a *nontrivial* δ-*derivation* we mean a nonzero δ-derivation, which is neither a 1-derivation nor a 0-derivation. Clearly, the multiplication operator by an element of the ground field F is a $\frac{1}{2}$-derivation in every superalgebra. We are interested in studying the action of nontrivial δ-derivations on classical Lie superalgebras over an algebraically closed field F of characteristic 0.

§ 2. The Main Lemmas

In this section, we formulate and prove the main lemmas, which will be used in subsequent investigation of the action of δ-derivations of classical Lie superalgebras.

Lemma 3. Let ϕ be a nontrivial δ-derivation of a Lie superalgebra G and $x \in G_1$. Then $\phi(x^2) \in G_0$.

Proof. Let $\phi(x) = x_0 + x_1, x_i \in G_i$. Then

$$\phi(x^2) = \delta([\phi(x), x] + [x, \phi(x)]) = \delta([x_0 + x_1, x] + [x, x_0 + x_1]) = 2\delta[x_1, x] \in G_0.$$

The lemma is proved.

Lemma 4. Let ϕ be a nontrivial δ-derivation of a classical Lie superalgebra $G = G_0 + G_1$. Then $\phi(G_0) \subseteq G_0$.

Proof. In [5], it was shown that $[G_1, G_1] = G_0$. Then, for an arbitrary $x \in G_0$, $x = \sum\limits_{i=1}^{n_x} y_i z_i$, where $y_i, z_i \in G_1$. It is easy to see that $x = \sum\limits_{i=1}^{n_x} \big(\frac{1}{4}(y_i + z_i)^2 - \frac{1}{4}(y_i - z_i)^2\big)$. Now, we obtain the required assertion by Lemma 3. The lemma is proved.

Lemma 5. Let ϕ be a nontrivial δ-derivation of a simple Lie algebra G and $\dim(G) \geq 3$ over a field F of characteristic 0. Then either $\delta = \frac{1}{2}$ and there exists $\alpha \in F$ such that $\phi(x) = \alpha x$ for every $x \in G$ or $\delta = -1$ and $G \cong A_1$.

Proof. The required assertion follows from Theorems 3, 5, 6 in [2].

Express the Lie algebra A_1 in the shape of the algebra of columns of order 3 with the product as follows:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} bx - cy \\ 2ay - 2bx \\ 2cx - 2az \end{bmatrix}, \text{ где } \begin{bmatrix} a & b \\ c & -a \end{bmatrix} \mapsto \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

Denote by Antider(A_1) the space of antiderivations of A_1. Then is valid the following

Lemma 6. $Antider(A_1) = \left\{ \begin{bmatrix} -2a & b & c \\ 2c & a & d \\ 2b & e & a \end{bmatrix} : a, b, c, d, e \in F \right\}$.

Proof. The required assertion follows from [1].

Lemma 7. Let ϕ be a nontrivial δ-derivation of a Lie algebra $L = L_1 \oplus L_2$, where L_1 is a semisimple Lie algebra. Then $\phi(L_2) \subseteq L_2$.

Proof. Let $x_i \in L_i$ and $\phi(x_i) = x_i^1 + x_i^2$, where $x_i^j \in L_j$. Then we have

$$0 = \phi([x_1, x_2]) = \delta\big(\big[x_1, x_2^1 + x_2^2\big] + \big[x_1^1 + x_1^2, x_2\big]\big) = \delta\big(\big[x_1, x_2^1\big] + \big[x_1^2, x_2\big]\big),$$

whence $\big[x_1, x_2^1\big] = 0$. It is known that the center of a semisimple Lie algebra is zero. Therefore, $x_2^1 = 0$ and $\phi(L_2) \subseteq L_2$. The lemma is proved.

Lemma 8. Let $G = G_0 + G_1$ be a basic classical Lie superalgebra, and let ϕ be a nontrivial δ-derivation of G. Then $\phi(G_1) \subseteq G_1$.

Proof. Let H be a Cartan subalgebra in G. Take $g_\beta \in G_1 \cap G_\beta$ and $h \in H$. Then $\phi(g_\beta) = \sum\limits_{\gamma\in\Delta} k_\gamma^\beta g_\gamma$, and $\sum\limits_{\gamma\in\Delta} \beta(h) k_\gamma^\beta g_\gamma = \phi(\beta(h) g_\beta) = \phi(h g_\beta) = \delta(\phi(h) g_\beta + h\phi(g_\beta)) = \delta(\phi(h) g_\beta + \sum\limits_{\gamma\in\Delta} \gamma(h) k_\gamma^\beta g_\gamma)$. By $\phi(h) g_\beta \in G_1$ and the arbitrariness of $h \in H$, we have $\beta = \delta\gamma$ when $\gamma \in \Delta_0$ and $k_\gamma^\beta \neq 0$. If $\delta \neq \frac{1}{2}$ then the lemma is proved. If $\delta = \frac{1}{2}$ then $\phi(g_\beta) = g_1^\beta + g_{2\beta}$ and $\phi(g_{-\beta}) = g_1^{-\beta} + g_{-2\beta}$, where $g_1^\beta, g_1^{-\beta} \in G_1$, $g_{2\beta}, g_{-2\beta} \in G_0$. Then $\frac{1}{2}\big(g_{2\beta} g_{-\beta} + g_1^\beta g_{-\beta} + g_\beta g_1^{-\beta} + g_\beta g_{-2\beta}\big) = \phi(g_\beta g_{-\beta}) \in G_0$, i. e., $g_{2\beta} = g_{-2\beta} = 0$, whence $\phi(g_\beta) \in G_1$. The lemma is proved.

§ 3. δ-Derivations of Classical Lie Superalgebras

The majority of this section concerns with the action of antiderivations on Lie superalgebras, which possess a direct summand in the even part either of type A_1 or F, where F is the ground field. In [1], it was shown that A_1 admits a nontrivial antiderivation. It is clear that if we consider a field F with respect to the Lie multiplication then F admits a nontrivial antiderivation, that acts as follows: $\phi(f) = \alpha f$, where $f, \alpha \in F$. The remaining part of this section is devoted to δ-derivations of classical Lie superalgebras, which are trivial on the even part, and to the action of nontrivial $\frac{1}{2}$-derivations on classical Lie superalgebras.

Lemma 9. Let $G = G_0 + G_1$ be a basic classical Lie superalgebra, let ϕ be a δ-derivation of G such that $\phi(G_0) = 0$. Then ϕ is trivial.

Proof. Let $G = \bigoplus\limits_{\beta\in\Delta} G_\beta$ be the root decomposition with respect to a Cartan subalgebra H. Take $h \in H$. By $H \subseteq G_0$, we have $\phi(h) = 0$. Then, for $g_\alpha \in G_1 \cap G_\alpha$, we obtain $\alpha(h)\phi(g_\alpha) = \phi(h g_\alpha) = \delta(\phi(h) g_\alpha + h\phi(g_\alpha)) = \delta h \phi(g_\alpha)$. By the arbitrariness of h, $\phi(g_\alpha) \in G_{\frac{\alpha}{\delta}}$, whence $\delta = \pm 1, \pm\frac{1}{2}$ by the property of roots.

The case $\delta = 1$ gives an ordinary derivation, i. e., ϕ is trivial.

For $\delta = \frac{1}{2}$, we have $\phi(g_\alpha) = g_{2\alpha}$, $\phi(g_{-\alpha}) = g_{-2\alpha}$. Thus, $0 = \phi(g_\alpha g_{-\alpha}) = \frac{1}{2}(g_{2\alpha} g_{-\alpha} + g_\alpha g_{-2\alpha})$. It is easy to see that $g_{2\alpha} g_{-\alpha} = 0$. Hence, $\phi(g_\alpha) = 0$.

If $\delta = -\frac{1}{2}$ then $\phi(g_\alpha) = g_{-2\alpha}$. If 2α is not a root then $\phi(g_\alpha) = 0$. If 2α is a root then $\phi(g_{-\alpha}) = g_{2\alpha}^*$, $\phi(g_{2\alpha}) = 0$, and $\phi(g_\alpha) = \phi(g_{2\alpha} g_{-\alpha}) = -\frac{1}{2}(g_{2\alpha} g_{2\alpha}^*) = 0$, which gives $g_{2\alpha}^* = 0$ and $\phi(g_\alpha) = 0$.

In the case $\delta = -1$, we need consider every classical Lie superalgebra separately. In this case, for $g_\beta \in G_\beta$, $g_\beta \in G_1$ and some $g_{-\beta} \in G_{-\beta}$, we have $\phi(g_\beta) = g_{-\beta}$. It suffices to show that $\phi(G_\beta) = 0$ for $\beta \in \Pi$.

The case $A(m,n)$:

$$\phi(g_{\epsilon_{m+1}-\delta_1}) = \phi(g_{\epsilon_{m+1}-\delta_2} g_{\delta_2-\delta_1}) = -\phi(g_{\epsilon_{m+1}-\delta_2}) g_{\delta_2-\delta_1} = -g_{-\epsilon_{m+1}+\delta_2} g_{\delta_2-\delta_1} = 0.$$

The other cases may be considered analogously. Therefore, $\phi(G) = 0$ by the property of simple system of roots. The lemma is proved.

Lemma 10 Let $G = G_0 + G_1$ be a basic classical Lie superalgebra, where $G_0 = G_s \oplus F$, and let ϕ be a δ-derivation of G such that $\phi(G_s) = 0$. Then ϕ is trivial.

Proof. Let $G = \bigoplus\limits_{\beta\in\Delta} G_\beta$ be the root decomposition with respect to a Cartan subalgebra H. Take an arbitrary h in H. It is clear that $\phi(h) = \alpha h$ when $h \in F$. Take $g_\beta \in G_1 \cap G_\beta$. Then

$\phi(g_\beta) = \sum\limits_{\gamma\in\Delta} k_\gamma g_\gamma$, and we have

$$\beta(h)\sum_{\gamma\in\Delta} k_\gamma g_\gamma = \phi(hg_\beta) = \delta(\phi(h)g_\beta + h\phi(g_\beta)) = \delta\Big(\alpha\beta(h)g_\beta + \sum_{\gamma\in\Delta} k_\gamma \gamma(h) g_\gamma\Big),$$

whence $k_\gamma \neq 0$ when $\beta = \delta\gamma$, and $\alpha = \frac{1-\delta}{\delta}k_\beta$. Hence, $\phi(g_\beta) = k_\beta g_\beta + k_{\frac{\beta}{\delta}} g_{\frac{\beta}{\delta}}$. Thus, if $\delta \neq -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$ then $\phi(g_\beta) = k_\beta g_\beta$.

If $\delta = \frac{1}{2}$ then $\phi(g_\beta) = k_\beta g_\beta + k_{2\beta} g_{2\beta}, \phi(g_{-\beta}) = k_{-\beta} g_{-\beta} + k_{-2\beta} g_{-2\beta}$, which gives $k_{2\beta} g_{2\beta} g_{-\beta} + k_{-2\beta} g_\beta g_{-2\beta} + k_\beta g_{-\beta} g_\beta + k_{-\beta} g_\beta g_{-\beta} = \phi(g_\beta g_{-\beta}) \in G_0$, i. e., $\phi(g_\beta) = k_\beta g_\beta$.

If $\delta = -1, -\frac{1}{2}$ then $0 = \phi(g_\beta g_\beta) = -2(k_\beta g_\beta + k_{\frac{\beta}{\delta}} g_{\frac{\beta}{\delta}}) g_\beta$, whence $k_{\frac{\beta}{\delta}} = 0$.

Now, it is clear that $\phi(g_\beta) = \frac{\delta}{1-\delta}\alpha g_\beta$. For $x \in G_s$, we have $x = \sum\limits_{i=1}^{n_x} x_i y_i$, where $x_i, y_i \in G_1$. Then

$$0 = \phi(x) = \phi\left(\sum_{i=1}^{n_x} x_i y_i\right) = \delta\left(\sum_{i=1}^{n_x}(\phi(x_i)y_i + x_i\phi(y_i))\right) = \frac{2\delta^2\alpha}{1-\delta}\sum_{i=1}^{n_x} x_i y_i = \frac{2\delta^2\alpha}{1-\delta}x,$$

which implies $\alpha = 0$, i. e., we arrive at the triviality of ϕ. The lemma is proved.

Lemma 11. Let $G = G_0 + G_1$ be a basic classical Lie superalgebra, and let ϕ be a nontrivial $\frac{1}{2}$-derivation of G. Then $\phi(x) = \alpha x$ for an arbitrary $x \in G$ and some $\alpha \in F$.

Proof. Let ϕ be a nontrivial $\frac{1}{2}$-derivation of G. By Theorem 1, $G_0 = G^1 \oplus G^2 \oplus G^3$, where G^i is a simple Lie algebra (some of G^i may be zero). By Lemma 5, $\phi(x) = \alpha_i x$ with $\alpha_i \in F$ and $x \in G^i$. In particular, $\phi(h^i) = \alpha_i h^i$ for $h^i \in H \cap G^i$. Then for $g_\beta \in G_\beta \cap G_1$ we may assume that $\phi(g_\beta) = \sum\limits_{\gamma\in\Delta_1} k_\gamma^\beta g_\gamma$, whence

$$\begin{aligned}\sum_{\gamma\in\Delta_1} k_\gamma^\beta \beta(h^i) g_\gamma &= \beta(h^i)\phi(g_\beta) = \phi(h^i g_\beta) = \\ \frac{1}{2}(h^i\phi(g_\beta) + \phi(h^i)g_\beta) &= \frac{\alpha_i + k_\beta^\beta}{2}\beta(h^i)g_\beta + \frac{1}{2}\sum_{\gamma\in\Delta_1, \gamma\neq\beta} k_\gamma^\beta \gamma(h^i) g_\gamma,\end{aligned}$$

i. e., $\alpha = \alpha_i = k_\beta^\beta$, $i = 1, 2, 3$. Replacing h^i by an arbitrary $h \in H$ in the obtained equality, we infer that $k_\gamma^\beta \neq 0$ with $\beta \neq \gamma$ only in the case $\gamma = 2\beta$. The latter is impossible because of $g_{2\beta} = (\theta g_\beta)^2 \in G_0$, $\theta \in F$, which contradicts Lemma 8. Thus, $\phi(x) = \alpha x, \alpha \in F$, for an arbitrary $x \in G$.

Lemma 12. The superalgebra $A(m, 1)$ with $m \neq 1$ does not admit nonzero antiderivations.

Proof. Assume that ϕ is a nontrivial antiderivation of $A(m, 1)$, $m \neq 1$. It is clear that $(A(m, 1))_0 = A_m \oplus A_1 \oplus F$. By Lemmas 4–7 we have $\phi(A_1) \subseteq A_1$, $\phi(A_m) = 0$, $\phi((A(m, 1))_1) \subseteq (A(m, 1))_1$, $\phi(F) \subseteq F$. Clearly, an antiderivation of F is the multiplication by an element of F.

Let $m \geq 2$. Fix the basis

$$g_{\epsilon_i-\delta_1} = e_{i,n+1}, \quad g_{\epsilon_i-\delta_2} = e_{i,n+2}, \quad g_{-\epsilon_i+\delta_1} = e_{n+1,i}, \quad g_{-\epsilon_i+\delta_2} = e_{n+2,i}.$$

The standard basis for A_1 is $\{h, g_{\delta_1-\delta_2}, g_{\delta_2-\delta_1}\}$. By Lemma 6, an antiderivation ϕ on A_1 looks as follows:

$$\phi(h) = -2ah + bg_{\delta_1-\delta_2} + cg_{\delta_2-\delta_1},$$

$$\phi(g_{\delta_1-\delta_2}) = 2ch + ag_{\delta_1-\delta_2} + dg_{\delta_2-\delta_1}, \quad \phi(g_{\delta_2-\delta_1}) = 2bh + eg_{\delta_1-\delta_2} + ag_{\delta_2-\delta_1}.$$

Let

$$\phi(g_{\epsilon_k-\delta_1}) = \sum_{j=1}^{m}\big(t_k^{\pm(\epsilon_j-\delta_1)} g_{\pm(\epsilon_j-\delta_1)} + t_k^{\pm(\epsilon_j-\delta_2)} g_{\pm(\epsilon_j-\delta_2)}\big).$$

Then

$$\phi(g_{\epsilon_i-\delta_1}) = \phi(g_{\epsilon_k-\delta_1} g_{\epsilon_i-\epsilon_k}) = -\sum_{j=1}^{m}(t_k^{\pm(\epsilon_j-\delta_1)} g_{\pm(\epsilon_j-\delta_1)} + t_k^{\pm(\epsilon_j-\delta_2)} g_{\pm(\epsilon_j-\delta_2)}) g_{\epsilon_i-\epsilon_k} =$$
$$-(t_k^{-\epsilon_i+\delta_1} g_{-\epsilon_i+\delta_1} g_{\epsilon_i-\epsilon_k} + t_k^{\epsilon_k-\delta_1} g_{\epsilon_k-\delta_1} g_{\epsilon_i-\epsilon_k} + t_k^{\epsilon_k-\delta_2} g_{\epsilon_k-\delta_2} g_{\epsilon_i-\epsilon_k} + t_k^{-\epsilon_i+\delta_2} g_{-\epsilon_i+\delta_2} g_{\epsilon_i-\epsilon_k}),$$

whence

$$\phi(g_{\epsilon_i-\delta_1}) = t_i^{\epsilon_i-\delta_1} g_{\epsilon_i-\delta_1} + t_i^{\epsilon_i-\delta_2} g_{\epsilon_i-\delta_2}$$

by the arbitrariness of k. Analogously,

$$\phi(g_{\pm(\epsilon_i-\delta_l)}) = k^{\pm(\epsilon_i-\delta_l)} g_{\pm(\epsilon_i-\delta_l)} + t^{\pm(\epsilon_i-\delta_{l+1})} g_{\pm(\epsilon_i-\delta_{l+1})}.$$

Here and further in the lemma, $l+1$ is considered modulo 2.

Let $(g_{\epsilon_i-\delta} g_{-\epsilon_i+\delta})|_F = f$ and $\phi(f) = \alpha f$ for some $\alpha \in F$. Then

$$\alpha f = \phi(f) = \phi(g_{\epsilon_i-\delta_l} g_{-\epsilon_i+\delta_l}) =$$
$$-(k^{\epsilon_i-\delta_l} g_{\epsilon_i-\delta_l} + t^{\epsilon_i-\delta_{l+1}} g_{\epsilon_i-\delta_{l+1}}) g_{-\epsilon_i+\delta_l} - g_{\epsilon_i-\delta_l}(k^{-\epsilon_i+\delta_l} g_{-\epsilon_i+\delta_l} + t^{-\epsilon_i+\delta_{l+1}} g_{-\epsilon_i+\delta_{l+1}}),$$

whence $t^{\pm(\epsilon_i-\delta_l)} = 0$ and $\phi(g_{\pm(\epsilon_i-\delta_l)}) = k^{\pm(\epsilon_i-\delta_l)} g_{\pm(\epsilon_i-\delta_l)}$, $\alpha = -(k^{\epsilon_i-\delta_l} + k^{-\epsilon_i+\delta_l})$.

It is clear that the subalgebra $B = \mathrm{span}\langle h, g_{\delta_1-\delta_2}, g_{\delta_2-\delta_1}, g_{\pm(\epsilon_1-\delta_1)}, g_{\pm(\epsilon_1-\delta_2)}\rangle$ is invariant under ϕ, and B is of type $A(0,1)$. Take $\epsilon = \epsilon_1$. Show that the antiderivations are trivial on $A(0,1)$.

Taking into account $g^2_{\pm(\epsilon-\delta_l)} = 0$ and using $0 = \phi\big(g^2_{\pm(\epsilon-\delta_l)}\big) = -2g_{\pm(\epsilon-\delta_l)}\phi(g_{\pm(\epsilon-\delta_l)})$, we easily obtain $\phi(g_{\pm(\epsilon-\delta_l)}) = k^{\pm(\epsilon-\delta_l)} g_{\pm(\epsilon-\delta_l)} + l^{\pm(\epsilon-\delta_{l+1})} g_{\pm(\epsilon-\delta_{l+1})}$. Let $(g_{\epsilon-\delta_l} g_{-\epsilon+\delta_l})|_F = f_l$. Then $\alpha f_l = \phi(f_l) = \phi(g_{\epsilon-\delta_l} g_{-\epsilon+\delta_l}) = -(k^{\epsilon-\delta_l} + k^{-\epsilon+\delta_l}) f_l - l^{\epsilon-\delta_l} g_{\epsilon-\delta_{l+1}} g_{-\epsilon+\delta_l} - l^{-\epsilon+\delta_{l+1}} g_{\epsilon-\delta_l} g_{-\epsilon+\delta_{l+1}}$, whence $\alpha = -(k^{\epsilon-\delta_l} + k^{-\epsilon+\delta_l})$, $l^{\pm(\epsilon-\delta_l)} = 0$.

By $g_{\delta_2-\delta_1} = g_{\epsilon-\delta_1} g_{-\epsilon+\delta_2}$ and Lemma 6 we have $2bh + eg_{\delta_1-\delta_2} + ag_{\delta_2-\delta_1} = \phi(g_{\delta_2-\delta_1}) = \phi(g_{\epsilon-\delta_1} g_{-\epsilon+\delta_2}) = -(k^{\epsilon-\delta_1} + k^{-\epsilon+\delta_2}) g_{\epsilon-\delta_1} g_{-\epsilon+\delta_2}$. Analogously, $2ch + ag_{\delta_1-\delta_2} + dg_{\delta_2-\delta_1} = \phi(g_{\delta_1-\delta_2}) = \phi(g_{\epsilon-\delta_2} g_{-\epsilon+\delta_1}) = -(k^{\epsilon-\delta_2} + k^{-\epsilon+\delta_1}) g_{\epsilon-\delta_2} g_{-\epsilon+\delta_1}$. Thus, $b = c = e = d = 0, \alpha = a = -\frac{1}{2}k^{\pm(\epsilon-\delta_l)}$.

Note that

$$-2ag_{-\epsilon+\delta_1} = \phi(g_{-\epsilon+\delta_1}) = \phi(hg_{-\epsilon+\delta_1}) = -\phi(h)g_{-\epsilon+\delta_1} - h\phi(g_{-\epsilon+\delta_1}) = 4ag_{-\epsilon+\delta_1},$$

i. e., $a = 0$. This implies the triviality of ϕ on $A_1 \oplus F$, i. e., ϕ is trivial on $(A(m,1))_0$, $m \neq 1$. By Lemma 9, we arrive at the required assertion. The lemma is proved.

Lemma 13. The superalgebra $B(m,1)$ does not admit nonzero antiderivations.

Proof. Assume that ϕ is a nontrivial antiderivation of $B(m,1)$. It is clear that $(B(m,1))_0 = B_m \oplus A_1$. By Lemmas 4–7 we have $\phi(A_1) \subseteq A_1, \phi(B_m) = 0$, $\phi((B(m,1))_1) \subseteq (B(m,1))_1$.

Let $m > 0$,

$$g_\delta = g_{\pm\epsilon_i+\delta} g_{\mp\epsilon_i}, \quad \phi(g_{\pm\epsilon_i+\delta}) = k^{\delta}_{i,\pm} g_\delta + k^{-\delta}_{i,\pm} g_{-\delta} + \sum_{j=1}^{m} k^{\pm\epsilon_j\pm\delta}_{i,\pm} g_{\pm\epsilon_j\pm\delta}.$$

Consequently,

$$\phi(g_\delta) = \phi(g_{\pm\epsilon_i+\delta}g_{\mp\epsilon_i}) = -\left(k_{i,\pm}^{\delta}g_\delta + k_{i,\pm}^{-\delta}g_{-\delta} + \sum_{j=1}^{m} k_{i,\pm}^{\pm\epsilon_j\pm\delta}g_{\pm\epsilon_j\pm\delta}\right)g_{\mp\epsilon_i}.$$

Consecutively choosing the sign $+$ or $-$ in $g_{\pm\epsilon_i+\delta}$, we obtain $\phi(g_\delta) \in \operatorname{span}\langle g_\delta, g_{-\delta}\rangle$ by the arbitrariness of i. Analogously, $\phi(g_{-\delta}) \in \operatorname{span}\langle g_\delta, g_{-\delta}\rangle$. These inclusions and $\phi(A_1) \subseteq A_1$ imply that $B = \operatorname{span}\langle h, g_{-\delta}, g_\delta, g_{2\delta}, g_{-2\delta}\rangle$ since $B(m,1)$ is invariant with respect to ϕ. Clearly, B is of type $B(0,1)$. Consider the action of ϕ on $B(0,1)$, and show its triviality.

Choose the classical basis in B: $h = e_{22} - e_{33}$, $g_{-2\delta} = e_{32}$, $g_{2\delta} = e_{23}$, $g_{-\delta} = e_{12} - e_{31}$, $g_\delta = e_{13} + e_{21}$. By Lemma 6, we have

$$\phi(h) = -2ah + bg_{2\delta} + cg_{-2\delta},\ \phi(g_{2\delta}) = 2ch + ag_{2\delta} + dg_{-2\delta},\ \phi(g_{-2\delta}) = 2bh + eg_{2\delta} + ag_{-2\delta}.$$

Let $\phi(g_\delta) = kg_\delta + lg_{-\delta}$, $\phi(g_{-\delta}) = k^*g_\delta + l^*g_{-\delta}$. Then

$$kg_\delta + lg_{-\delta} = \phi(g_\delta) = \phi(hg_\delta) = -(-2ah + bg_{2\delta} + cg_{-2\delta})g_\delta - h(kg_\delta + lg_{-\delta}) = 2ag_\delta + cg_{-\delta} - kg_\delta + lg_{-\delta},$$

whence $a = k$, $c = 0$. Analogously,

$$-k^*g_\delta - l^*g_{-\delta} = -\phi(g_{-\delta}) = \phi(hg_{-\delta}) = -(-2ah + bg_{2\delta})g_{-\delta} - h(k^*g_\delta + l^*g_{-\delta}) = -2ag_{-\delta} + bg_\delta - k^*g_\delta + l^*g_{-\delta},$$

which gives $a = l^*, b = 0$.

Note that

$$2(ag_{2\delta} + dg_{-2\delta}) = \phi(2g_{2\delta}) = \phi(g_\delta g_\delta) = -2g_\delta(ag_\delta + lg_{-\delta}) = -4ag_{2\delta} - 2lh.$$

Hence, $a = l = d = 0$. It remains to remark that $-2eg_{2\delta} = -2\phi(g_{-2\delta}) = \phi(g_{-\delta}g_{-\delta}) = -2g_{-\delta}\phi(g_{-\delta}) = -2k^*g_{-\delta}g_\delta = -2k^*h$, whence $e = k^* = 0$. Thus, ϕ is trivial on A_1, i. e., ϕ is trivial on $(B(m,1))_0$. Lemma 9 gives the required assertion. The lemma is proved.

Lemma 14. The superalgebra $C(n)$ does not admit nonzero antiderivations.

Proof. Assume that ϕ is a nontrivial antiderivation of $C(n)$. Clearly, $(C(n))_0 = F \oplus C_{n-1}$. By Lemmas 4–7, we have $\phi(F) \subseteq F$, $\phi(C_n) = 0$, $n \geq 2$, $\phi(C_1) \subseteq C_1 (C_1 = A_1)$, $\phi((C(n))_1) \subseteq (C(n))_1$.

Assume that $\phi(g_{\epsilon+\delta_i}) = \sum\limits_{j=1}^{n-1} l_i^{\pm\epsilon\pm\delta_j}g_{\pm\epsilon\pm\delta_j}$. Then

$$0 = \phi(g_{\epsilon+\delta_i}^2) = -2\left(\sum_{j=1}^{n-1} l_i^{\pm\epsilon\pm\delta_j}g_{\pm\epsilon\pm\delta_j}\right)g_{\epsilon+\delta_i},$$

whence $\phi(g_{\epsilon+\delta_i}) = \sum\limits_{j=1}^{n-1} l_i^{\epsilon\pm\delta_j}g_{\epsilon\pm\delta_j}$, and analogously $\phi(g_{-\epsilon-\delta_i}) = \sum\limits_{j=1}^{n-1} m_i^{-\epsilon\pm\delta_j}g_{-\epsilon\pm\delta_j}$.

In the case $n \geq 3$, for $(g_{\epsilon+\delta_i}g_{-\epsilon-\delta_i})|_F = \alpha(e_{11} - e_{22})$, $\alpha \neq 0$ and $\phi(e_{11} - e_{22}) = k(e_{11} - e_{22})$, we have

$$\alpha k(e_{11} - e_{22}) = \alpha\phi(e_{11} - e_{22}) = \phi(g_{\epsilon+\delta_i}g_{-\epsilon-\delta_i}) = -(g_{\epsilon+\delta_i}\sum_{j=1}^{n-1} m_i^{-\epsilon\pm\delta_j}g_{-\epsilon\pm\delta_j} + \sum_{j=1}^{n-1} l_i^{\epsilon\pm\delta_j}g_{\epsilon\pm\delta_j}g_{-\epsilon-\delta_i}).$$

By (1), $(g_{\epsilon+\delta_i} g_{-\epsilon-\delta_i})|_{C_{n-1}} \neq 0$, whence

$$\alpha k(e_{11}-e_{22}) = \left(m_i^{-\epsilon\pm\delta_i} + l_i^{\epsilon+\delta_i}\right) g_{\epsilon+\delta_i} g_{-\epsilon-\delta_i},$$

i. e., $k=0$, and ϕ is trivial on the odd part of $C(n)$ when $n \geq 3$.

We consider the case $n=2$ in detail. Assume that $\delta=\delta_1$ and $\phi(g_{\pm\epsilon\pm\delta}) = k^{\pm\epsilon\pm\delta} g_{\pm\epsilon\pm\delta} + l^{\pm\epsilon\pm\delta} g_{\pm\epsilon\mp\delta}$. Fix the basis

$$g_{\epsilon-\delta} = e_{13}-e_{42}, \quad g_{\epsilon+\delta} = e_{14}+e_{32}, \quad g_{-\epsilon+\delta} = e_{24}+e_{31}, \quad g_{-\epsilon-\delta} = e_{23}-e_{41}.$$

We have

$$\begin{aligned} 4ch + 2ag_{2\delta} + 2dg_{-2\delta} &= \phi(2g_{2\delta}) = \phi(g_{\epsilon+\delta} g_{-\epsilon+\delta}) = -(k^{\epsilon+\delta} g_{\epsilon+\delta} + l^{\epsilon+\delta} g_{\epsilon-\delta}) g_{-\epsilon+\delta} - \\ g_{\epsilon+\delta}(k^{-\epsilon+\delta} g_{-\epsilon+\delta} + l^{-\epsilon+\delta} g_{\epsilon+\delta}) &= -2k^{\epsilon+\delta} g_{2\delta} - (l^{\epsilon+\delta} + l^{-\epsilon+\delta})h - 2k^{-\epsilon+\delta} g_{2\delta}, \end{aligned}$$

whence $d=0, 4c = -l^{\epsilon+\delta} - l^{-\epsilon+\delta}$.

Analogously, for $\phi(g_{-2\delta})$, we obtain

$$-(4bh + 2eg_{2\delta} + 2ag_{-2\delta}) = -\phi(2g_{-2\delta}) = \phi(g_{-\epsilon-\delta} g_{\epsilon-\delta}) = 2k^{-\epsilon-\delta} g_{-2\delta} - (l^{-\epsilon-\delta} + l^{\epsilon-\delta})h + 2k^{\epsilon-\delta} g_{-2\delta},$$

whence $d=0, 2c = -l^{-\epsilon-\delta} - l^{\epsilon-\delta}$.

Now, notice that for $\phi(h)$ hold

$$\begin{aligned} -2ah + bg_{2\delta} + cg_{-2\delta} &= \phi(h) = \phi(g_{\epsilon+\delta} g_{-\epsilon-\delta}) = -(k^{\epsilon+\delta} g_{\epsilon+\delta} + l^{\epsilon+\delta} g_{\epsilon-\delta}) g_{-\epsilon-\delta} - \\ g_{\epsilon+\delta}(k^{-\epsilon-\delta} g_{-\epsilon-\delta} + l^{-\epsilon-\delta} g_{-\epsilon+\delta}) &= -(k^{\epsilon+\delta} + k^{-\epsilon-\delta}) g_{\epsilon+\delta} g_{-\epsilon-\delta} + 2l^{\epsilon+\delta} g_{-2\delta} - 2l^{-\epsilon-\delta} g_{2\delta}. \end{aligned}$$

Using $(g_{\epsilon+\delta} g_{-\epsilon-\delta})|_F \neq 0$, we get $a=0, b=-2l^{-\epsilon-\delta}, c = 2l^{\epsilon+\delta}$. On the other hand, it is clear that

$$\begin{aligned} bg_{2\delta} + cg_{-2\delta} &= \phi(h) = \phi(g_{-\epsilon+\delta} g_{\epsilon-\delta}) = -(k^{-\epsilon+\delta} g_{-\epsilon+\delta} + l^{-\epsilon+\delta} g_{-\epsilon-\delta}) g_{\epsilon-\delta} - \\ g_{-\epsilon+\delta}(k^{\epsilon-\delta} g_{\epsilon-\delta} + l^{\epsilon-\delta} g_{\epsilon+\delta}) &= 2l^{-\epsilon+\delta} g_{-2\delta} - 2l^{\epsilon-\delta} g_{2\delta}, \end{aligned}$$

i. e., $c = 2l^{-\epsilon+\delta}, b = -2l^{\epsilon-\delta}$.

Comparing the obtained results, we have $4b = -l^{-\epsilon-\delta} - l^{\epsilon-\delta} = b$ and $4c = -l^{\epsilon+\delta} - l^{-\epsilon+\delta} = -c$, whence $b=c=0$, i. e., ϕ is trivial on $(C(2))_0$. By lemma 9, we arrive at the required assertion. The lemma is proved.

Lemma 15. The superalgebra $D(n,1)$ does not admit nonzero antiderivations.

Proof. Assume that ϕ is a nontrivial antiderivation of $D(n,1)$. It is clear that $(D(n,1))_0 = D_n \oplus A_1$. By Lemmas 4–7, we have $\phi(A_1) \subseteq A_1$, $\phi(D_n) = 0$, $\phi((D(n,1))_1) \subseteq (D(n,1))_1$, and ϕ acts on A_1 by the standard way, i. e.,

$$\phi(h) = -2ah + bg_{2\delta} + cg_{-2\delta}, \ \phi(g_{2\delta}) = 2ch + ag_{2\delta} + dg_{-2\delta}, \ \phi(g_{-2\delta}) = 2bh + eg_{2\delta} + ag_{-2\delta}.$$

Let $\phi(g_{\epsilon_j+\delta}) = \sum\limits_{i=1}^{n} l_j^{\pm\epsilon_i\pm\delta} g_{\pm\epsilon_i\pm\delta}$. Then

$$\phi(g_{\epsilon_j+\delta}) = \phi(g_{\epsilon_i+\delta} g_{\epsilon_j-\epsilon_i}) = -\sum_{k=1}^{m} l_i^{\pm\epsilon_k\pm\delta} g_{\pm\epsilon_k\pm\delta} g_{\epsilon_j-\epsilon_i} = l_j^{\epsilon_j\pm\delta} g_{\epsilon_j\pm\delta} + l_j^{-\epsilon_i\pm\delta} g_{-\epsilon_i\pm\delta}.$$

It is easy to obtain $\phi(g_{-\epsilon_i-\delta}) = l_{-i}^{-\epsilon_i\pm\delta} g_{-\epsilon_i\pm\delta} + l_{-i}^{\epsilon_j\pm\delta} g_{\epsilon_j\pm\delta}$.

Note that $(g_{\epsilon_i+\delta}g_{-\epsilon_i-\delta})D_n \neq 0$ by (1). If $(g_{\epsilon_i+\delta}g_{-\epsilon_i-\delta})|_{A_1} = \alpha h$, $\alpha \in F$, then

$$\alpha\phi(h) = \phi(g_{\epsilon_i+\delta}g_{-\epsilon_i-\delta}) = -(l_i^{\epsilon_j\pm\delta}g_{\epsilon_j\pm\delta} + l_i^{-\epsilon_i\pm\delta}g_{\epsilon_i\pm\delta})g_{-\epsilon_i-\delta} - g_{\epsilon_i+\delta}(l_{-i}^{-\epsilon_i\pm\delta}g_{-\epsilon_i\pm\delta} + l_{-i}^{\epsilon_j\pm\delta}g_{\epsilon_j\pm\delta}),$$

whence $a = 0$ and $l_i^{-\epsilon_j+\delta} = 0$, since ϕ is invariant on A_1.

Since $g_{\epsilon_i+\delta}^2 = 0$, we have $0 = \phi(g_{\epsilon_i+\delta}^2) = -2l_i^{-\epsilon_j-\delta}g_{\epsilon_i+\delta}g_{-\epsilon_j-\delta}$. Thus, $l_i^{-\epsilon_j-\delta} = 0$, i. e., $\phi(g_{\epsilon_i+\delta}) = l_i^{\epsilon_i\pm\delta}g_{\epsilon_i\pm\delta}$. Analogously, we may show that

$$\phi(g_{-\epsilon_i+\delta}) = k_i^{-\epsilon_i\pm\delta}g_{-\epsilon_i\pm\delta}, \quad \phi(g_{-\epsilon_i-\delta}) = m_i^{-\epsilon_i\pm\delta}g_{-\epsilon_i\pm\delta}, \quad \phi(g_{\epsilon_i-\delta}) = p_i^{\epsilon_i\pm\delta}g_{\epsilon_i\pm\delta}.$$

Prove that $b = c = d = e = 0$. Fix the following basis elements

$$g_{\epsilon_i-\delta} = e_{i,2n+1} - e_{2n+2,n+i}, \quad g_{\epsilon_i+\delta} = e_{i,2n+2} + e_{2n+1,n+i},$$

$$g_{-\epsilon_i-\delta} = e_{n+i,2n+1} - e_{2n+2,i}, \quad g_{-\epsilon_i-\delta} = e_{n+i,2n+2} + e_{2n+1,i}.$$

We have

$$\begin{aligned} bg_{2\delta} + cg_{-2\delta} = \phi(h) = \phi(g_{\epsilon_i+\delta}g_{-\epsilon_i-\delta}) &= -(l_i^{\epsilon_i+\delta}g_{\epsilon_i+\delta} + l_i^{\epsilon_i-\delta}g_{\epsilon_i-\delta})g_{-\epsilon_i-\delta} - \\ g_{\epsilon_i+\delta}(m_i^{-\epsilon_i-\delta}g_{-\epsilon_i-\delta} + m_i^{-\epsilon_i+\delta}g_{-\epsilon_i+\delta}) &= -(l_i^{\epsilon_i+\delta} + m_i^{-\epsilon_i-\delta})h + 2l_i^{\epsilon_i-\delta}g_{-2\delta} - 2m_i^{-\epsilon_i+\delta}g_{2\delta}, \end{aligned}$$

whence $b = -2m_i^{\epsilon_i+\delta}$, $c = 2l_i^{\epsilon_i-\delta}$.

Analogously,

$$\begin{aligned} bg_{2\delta} + cg_{-2\delta} = \phi(h) = \phi(g_{-\epsilon_i+\delta}g_{\epsilon_i-\delta}) &= -(k_i^{-\epsilon_i+\delta}g_{-\epsilon_i+\delta} + k_i^{-\epsilon_i+\delta}g_{-\epsilon_i-\delta})g_{\epsilon_i-\delta} - \\ g_{-\epsilon_i+\delta}(p_i^{\epsilon_i-\delta}g_{\epsilon_i-\delta} + p_i^{\epsilon_i+\delta}g_{\epsilon_i+\delta}) &= -(k_i^{-\epsilon_i+\delta} + p_i^{\epsilon_i-\delta})h + 2k_i^{-\epsilon_i-\delta}g_{-2\delta} - 2p_i^{\epsilon_i+\delta}g_{2\delta}, \end{aligned}$$

which implies $b = -2p_i^{\epsilon_i+\delta}$, $c = 2k_i^{-\epsilon_i-\delta}$.

Thus,

$$\begin{aligned} 4ch + 2eg_{-2\delta} = \phi(2g_{2\delta}) = \phi(g_{\epsilon_i+\delta}g_{-\epsilon_i+\delta}) &= -(l_i^{\epsilon_i+\delta}g_{\epsilon_i+\delta} + l_i^{\epsilon_i-\delta}g_{\epsilon_i-\delta})g_{-\epsilon_i+\delta} - \\ g_{\epsilon_i+\delta}(k_i^{-\epsilon_i+\delta}g_{-\epsilon_i+\delta} + k_i^{-\epsilon_i-\delta}g_{-\epsilon_i-\delta}) &= -(2l_i^{\epsilon_i+\delta} + 2k_i^{-\epsilon_i+\delta})g_{2\delta} - (l_i^{\epsilon_i-\delta} + k_i^{-\epsilon_i-\delta})h. \end{aligned}$$

So, $4c = -\big(l_i^{\epsilon_i-\delta} + k_i^{-\epsilon_i-\delta}\big)$ and $e = 0$, which gives $4c = -\big(l_i^{\epsilon_i-\delta} + k_i^{-\epsilon_i-\delta}\big) = -c$, i. e., $c = 0$.

It remains to notice that

$$\begin{aligned} -4bh - dg_{2\delta} = -2\phi(g_{-2\delta}) = \phi(g_{-\epsilon_i-\delta}g_{-\epsilon_i-\delta}) &= -(p_i^{\epsilon_i-\delta}g_{\epsilon_i-\delta} + p_i^{\epsilon_i+\delta}g_{\epsilon_i+\delta})g_{-\epsilon_i-\delta} - \\ g_{\epsilon_i-\delta}(m_i^{-\epsilon_i-\delta}g_{-\epsilon_i-\delta} + m_i^{-\epsilon_i+\delta}g_{-\epsilon_i+\delta}) &= 2p_i^{\epsilon_i-\delta}g_{-2\delta} - (p_i^{\epsilon_i+\delta} + m_i^{-\epsilon_i+\delta})h + 2m_i^{-\epsilon_i-\delta}g_{-2\delta}, \end{aligned}$$

whence $4b = p_i^{\epsilon_i+\delta} + m_i^{-\epsilon_i+\delta}$ and $d = 0$, which gives $4b = p_i^{\epsilon_i+\delta} + m_i^{-\epsilon_i+\delta} = -b$, i. e., $b = 0$.

It is clear that ϕ is trivial on $(D(n,1))_0$. Lemma 9 gives the required assertion. The lemma is proved.

Lemma 16. The superalgebra $D(2,1;\alpha)$ does not admit nonzero antiderivations.

Proof. Assume that ϕ is a nontrivial antiderivation of $D(2,1;\alpha)$. It is clear that $(D(2,1;\alpha))_1 = A_1^1 \oplus A_1^2 \oplus A_1^3$, $A_1^j \cong A_1$. By Lemmas 4–7, we have $\phi\big(A_1^j\big) \subseteq A_1^j$, $\phi((D(2,1;\alpha))_1) \subseteq (D(2,1;\alpha))_1$.

Fix the basis $\{h_i, g_{2\epsilon_i}, g_{-2\epsilon_i}\}$ in A_1^i. The basis of the odd part looks as follows:

$$g_{\epsilon_1+\epsilon_2+\epsilon_3} = (1,0) \otimes (1,0) \otimes (1,0), \quad g_{-\epsilon_1+\epsilon_2+\epsilon_3} = (0,1) \otimes (1,0) \otimes (1,0),$$

$$g_{\epsilon_1-\epsilon_2+\epsilon_3}=(1,0)\otimes(0,1)\otimes(1,0),\quad g_{\epsilon_1+\epsilon_2-\epsilon_3}=(1,0)\otimes(1,0)\otimes(0,1),$$
$$g_{-\epsilon_1-\epsilon_2+\epsilon_3}=(0,1)\otimes(0,1)\otimes(1,0),\quad g_{-\epsilon_1+\epsilon_2-\epsilon_3}=(0,1)\otimes(1,0)\otimes(0,1),$$
$$g_{\epsilon_1-\epsilon_2-\epsilon_3}=(1,0)\otimes(0,1)\otimes(0,1),\quad g_{-\epsilon_1-\epsilon_2-\epsilon_3}=(0,1)\otimes(0,1)\otimes(0,1).$$

From here we see that $h_ig_{\epsilon_1+\epsilon_2+\epsilon_3}=g_{\epsilon_1+\epsilon_2+\epsilon_3}$, $g_{-2\epsilon_i}g_{\epsilon_1+\epsilon_2+\epsilon_3}=g_{\epsilon_1+\epsilon_2+\epsilon_3-2\epsilon_i}$.

Since $g^2_{\epsilon_1+\epsilon_2+\epsilon_3}=0$, we have $0=\phi\big(g^2_{\epsilon_1+\epsilon_2+\epsilon_3}\big)=-2\phi(g_{\epsilon_1+\epsilon_2+\epsilon_3})g_{\epsilon_1+\epsilon_2+\epsilon_3}$. Therefore,

$$\phi(g_{\epsilon_1+\epsilon_2+\epsilon_3})=kg_{\epsilon_1+\epsilon_2+\epsilon_3}+lg_{-\epsilon_1+\epsilon_2+\epsilon_3}+ng_{\epsilon_1-\epsilon_2+\epsilon_3}+mg_{\epsilon_1+\epsilon_2-\epsilon_3}.$$

It is easy to see that

$$\begin{array}{rcl}\phi(g_{\epsilon_1+\epsilon_2+\epsilon_3}) & = & \phi(h_1g_{\epsilon_1+\epsilon_2+\epsilon_3})=\\ -h_1(kg_{\epsilon_1+\epsilon_2+\epsilon_3}+lg_{-\epsilon_1+\epsilon_2+\epsilon_3}+ng_{\epsilon_1-\epsilon_2+\epsilon_3} & + & mg_{\epsilon_1+\epsilon_2-\epsilon_3})-(-2a^1h_1+b^1g_{2\epsilon_1}+cg_{-2\epsilon_1})g_{\epsilon_1+\epsilon_2+\epsilon_3}=\\ -kg_{\epsilon_1+\epsilon_2+\epsilon_3}+lg_{-\epsilon_1+\epsilon_2+\epsilon_3}-ng_{\epsilon_1-\epsilon_2+\epsilon_3} & - & mg_{\epsilon_1+\epsilon_2-\epsilon_3}+2a^1g_{\epsilon_1+\epsilon_2+\epsilon_3}-c^1g_{-\epsilon_1+\epsilon_2+\epsilon_3},\end{array}$$

i. e., $k=2a^1-k$, $l=l-c^1$, $n=m=0$, and $a^1=k$, $c^1=0$.

Analogously,

$$\begin{array}{rcl}\phi(g_{\epsilon_1+\epsilon_2+\epsilon_3}) & = & \phi(h^2g_{\epsilon_1+\epsilon_2+\epsilon_3})=-h^2(kg_{\epsilon_1+\epsilon_2+\epsilon_3}+lg_{-\epsilon_1+\epsilon_2+\epsilon_3})-\\ (-2a^2h^2+b^2g_{2\epsilon_2}+c^2g_{-2\epsilon_2})g_{\epsilon_1+\epsilon_2+\epsilon_3} & = & -kg_{\epsilon_1+\epsilon_2+\epsilon_3}-lg_{-\epsilon_1+\epsilon_2+\epsilon_3}+2a^2g_{\epsilon_1+\epsilon_2+\epsilon_3}-c^2g_{\epsilon_1-\epsilon_2+\epsilon_3},\end{array}$$

whence $k=a^2, c^2=l=0$. By an analogous argument for $\phi(h_3g_{\epsilon_1+\epsilon_2+\epsilon_3})$, we obtain

$$\begin{array}{rcl}\phi(g_{\epsilon_1+\epsilon_2+\epsilon_3})=\phi(h_3g_{\epsilon_1+\epsilon_2+\epsilon_3}) & = & -h_3(kg_{\epsilon_1+\epsilon_2+\epsilon_3})-(-2a^3h_3+b^2g_{2\epsilon_3}+c^2g_{-2\epsilon_3})g_{\epsilon_1+\epsilon_2+\epsilon_3}=\\ -kg_{\epsilon_1+\epsilon_2+\epsilon_3} & + & 2a^3g_{\epsilon_1+\epsilon_2+\epsilon_3}+c^3g_{\epsilon_1+\epsilon_2-\epsilon_3},\end{array}$$

which implies $a^3=k$, $c^3=0$.

An analogous argument for $g_{-\epsilon_1-\epsilon_2-\epsilon_3}$ gives $b^1=b^2=b^3=0$, $\phi(g_{-\epsilon_1-\epsilon_2-\epsilon_3})=ag_{-\epsilon_1-\epsilon_2-\epsilon_3}$, where $a=a^i$, $i=1,2,3$.

Now, show that d^i and e^i are zero. This fact follows from

$$\begin{array}{rcl}\lambda^i(ag_{2\epsilon_i}+d^ig_{-2\epsilon_i}) & = & \lambda^i\phi(g_{2\epsilon_i})=\\ \phi(g_{\epsilon_1+\epsilon_2+\epsilon_3}g_{-\epsilon_1-\epsilon_2-\epsilon_3+2\epsilon_i}) & = & -ag_{\epsilon_1+\epsilon_2+\epsilon_3}g_{-\epsilon_1-\epsilon_2-\epsilon_3+2\epsilon_i}-g_{\epsilon_1+\epsilon_2+\epsilon_3}\phi(g_{-\epsilon_1-\epsilon_2-\epsilon_3+2\epsilon_i}).\end{array}$$

Clearly, the right-hand side of this equality does not contain the elements of the shape $g_{-2\epsilon_i}$. Analogously,

$$\begin{array}{rcl}\mu^i(e^ig_{2\epsilon_i}+ag_{-2\epsilon_i}) & = & \mu^i\phi(g_{-2\epsilon_i})=\\ \phi(g_{\epsilon_1+\epsilon_2+\epsilon_3-2\epsilon_i}g_{-\epsilon_1-\epsilon_2-\epsilon_3}) & = & -ag_{\epsilon_1+\epsilon_2+\epsilon_3-2\epsilon_i}g_{-\epsilon_1-\epsilon_2-\epsilon_3}-\phi(g_{\epsilon_1+\epsilon_2+\epsilon_3-2\epsilon_i})g_{-\epsilon_1-\epsilon_2-\epsilon_3},\end{array}$$

where the right-hand side does not contain the elements of the shape $g_{2\epsilon_i}$ and $e^i=0$.

Now, we have

$$\phi(g_{\epsilon_1+\epsilon_2-\epsilon_3})=\phi(g_{-2\epsilon_3}g_{\epsilon_1+\epsilon_2+\epsilon_3})=-2ag_{\epsilon_1+\epsilon_2-\epsilon_3},$$
$$\phi(g_{\epsilon_1-\epsilon_2-\epsilon_3})=\phi(g_{-2\epsilon_2}g_{\epsilon_1+\epsilon_2-\epsilon_3})=ag_{\epsilon_1-\epsilon_2-\epsilon_3},$$
$$\phi(g_{-\epsilon_1-\epsilon_2-\epsilon_3})=\phi(g_{-2\epsilon_1}g_{\epsilon_1-\epsilon_2-\epsilon_3})=-2ag_{-\epsilon_1-\epsilon_2-\epsilon_3},$$

which implies $a = 0$, i. e., ϕ is trivial on $(D(2,1;\alpha))_0$. Lemma 9 gives the required assertion. The lemma is proved.

Lemma 17. The superalgebra $F(4)$ does not admit nonzero antiderivations.

Proof. Assume that ϕ is a nontrivial antiderivation of $F(4)$. It is clear that $(F(4))_0 = A_1 \oplus B_3$. By Lemmas 4–7, we have $\phi(A_1) \subseteq A_1, \phi(B_3) = 0, \phi((F(4))_1) \subseteq (F(4))_1$. Assume that ϕ acts on A_1 by the standard way, i. e., $\phi(h) = -2ah + bg_\delta + cg_{-\delta}$, $\phi(g_\delta) = 2ch + ag_\delta + dg_{-\delta}$, $\phi(g_{-\delta}) = 2bh + eg_\delta + ag_{-\delta}$.

It is easy to see that

$$0 = \phi\big(g^2_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}\big) = -2g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}\phi\big(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}\big),$$

whence

$$\begin{array}{rcl}\phi(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}) &=& l^{\epsilon_1+\epsilon_2+\epsilon_3+\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)} + l^{-\epsilon_1+\epsilon_2+\epsilon_3+\delta}g_{\frac{1}{2}(-\epsilon_1+\epsilon_2+\epsilon_3+\delta)} + \\ l^{\epsilon_1-\epsilon_2+\epsilon_3+\delta}g_{\frac{1}{2}(\epsilon_1-\epsilon_2+\epsilon_3+\delta)} &+& l^{\epsilon_1+\epsilon_2-\epsilon_3+\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2-\epsilon_3+\delta)} + l^{\epsilon_1+\epsilon_2+\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)} + \\ l^{-\epsilon_1-\epsilon_2+\epsilon_3+\delta}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2+\epsilon_3+\delta)} &+& l^{\epsilon_1-\epsilon_2-\epsilon_3+\delta}g_{\frac{1}{2}(\epsilon_1-\epsilon_2-\epsilon_3+\delta)} + l^{-\epsilon_1+\epsilon_2-\epsilon_3+\delta}g_{\frac{1}{2}(-\epsilon_1+\epsilon_2-\epsilon_3+\delta)}.\end{array}$$

Analogously,

$$\begin{array}{rcl}\phi(g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)}) &=& k^{-\epsilon_1-\epsilon_2-\epsilon_3-\delta}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} + k^{-\epsilon_1-\epsilon_2-\epsilon_3+\delta}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3+\delta)} + \\ k^{-\epsilon_1-\epsilon_2+\epsilon_3-\delta}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2+\epsilon_3-\delta)} &+& k^{-\epsilon_1+\epsilon_2-\epsilon_3-\delta}g_{\frac{1}{2}(-\epsilon_1+\epsilon_2-\epsilon_3-\delta)} + k^{\epsilon_1-\epsilon_2-\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1-\epsilon_2-\epsilon_3-\delta)} + \\ k^{\epsilon_1+\epsilon_2-\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2-\epsilon_3-\delta)} &+& k^{\epsilon_1-\epsilon_2+\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1-\epsilon_2+\epsilon_3-\delta)} + k^{-\epsilon_1+\epsilon_2+\epsilon_3-\delta}g_{\frac{1}{2}(-\epsilon_1+\epsilon_2+\epsilon_3-\delta)}.\end{array}$$

By (1), $(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)})|_{B_3} \neq 0$. Then if

$$(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)})|_{A_1} = \alpha h,$$

where $\alpha \neq 0$ (by (1)), then

$$\begin{array}{rcl}\alpha(-2ah + bg_\delta + cg_{-\delta}) = \alpha\phi(h) &=& \phi(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)}) = \\ -l^{\epsilon_1+\epsilon_2+\epsilon_3+\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} &-& l^{-\epsilon_1+\epsilon_2+\epsilon_3+\delta}g_{\frac{1}{2}(-\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} - \\ l^{\epsilon_1-\epsilon_2+\epsilon_3+\delta}g_{\frac{1}{2}(\epsilon_1-\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} &-& l^{\epsilon_1+\epsilon_2-\epsilon_3+\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2-\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} - \\ l^{\epsilon_1+\epsilon_2+\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} &-& l^{-\epsilon_1-\epsilon_2+\epsilon_3+\delta}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} - \\ l^{-\epsilon_1+\epsilon_2-\epsilon_3+\delta}g_{\frac{1}{2}(-\epsilon_1+\epsilon_2-\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} &-& l^{\epsilon_1-\epsilon_2-\epsilon_3+\delta}g_{\frac{1}{2}(\epsilon_1-\epsilon_2-\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} - \\ k^{-\epsilon_1-\epsilon_2-\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} &-& k^{\epsilon_1-\epsilon_2-\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(\epsilon_1-\epsilon_2-\epsilon_3-\delta)} - \\ k^{-\epsilon_1+\epsilon_2-\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1+\epsilon_2-\epsilon_3-\delta)} &-& k^{-\epsilon_1-\epsilon_2+\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2+\epsilon_3-\delta)} - \\ k^{-\epsilon_1-\epsilon_2-\epsilon_3+\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3+\delta)} &-& k^{\epsilon_1+\epsilon_2-\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(\epsilon_1+\epsilon_2-\epsilon_3-\delta)} - \\ k^{\epsilon_1-\epsilon_2+\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(\epsilon_1-\epsilon_2+\epsilon_3-\delta)} &-& k^{-\epsilon_1+\epsilon_2+\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1+\epsilon_2+\epsilon_3-\delta)},\end{array}$$

whence $a = 0$, and

$$\phi(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}) = k^{\epsilon_1+\epsilon_2+\epsilon_3+\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)} + k^{\epsilon_1+\epsilon_2+\epsilon_3-\delta}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)}.$$

Analogously,

$$\phi(g_{\frac{1}{2}(\pm\epsilon_1\pm\epsilon_2\pm\epsilon_3\pm\delta)}) = k^{\frac{1}{2}(\pm\epsilon_1\pm\epsilon_2\pm\epsilon_3\pm\delta)}g_{\frac{1}{2}(\pm\epsilon_1\pm\epsilon_2\pm\epsilon_3\pm\delta)} + l^{\frac{1}{2}(\pm\epsilon_1\pm\epsilon_2\pm\epsilon_3\mp\delta)}g_{\frac{1}{2}(\pm\epsilon_1\pm\epsilon_2\pm\epsilon_3\mp\delta)}.$$

If $g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3+\delta)} = \beta g_\delta$, $\beta \in F$, then

$$\begin{aligned}\beta(2ch + dg_{-\delta}) = \beta\phi(g_\delta) &= \phi(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3+\delta)}) = \\ -\phi(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)})g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3+\delta)} &- g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}\phi(g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3+\delta)}).\end{aligned}$$

It is clear that the right-hand side of this equality does not contain the elements of the shape $g_{-\delta}$. Thus, $d = 0$. Analogously, if $g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} = \gamma g_{-\delta}$, $\gamma \in F$, then the equalities

$$\begin{aligned}\gamma(2bh + eg_\delta) = \gamma\phi(g_{-\delta}) &= \phi(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)}g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)}) = \\ -\phi(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)})g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} &- g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)}\phi(g_{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)}),\end{aligned}$$

give $e = 0$.

Note that if $\alpha_1, \alpha_2 \in \Delta_1$, $\alpha_1 + \alpha_2 \neq 0$, and $\alpha_1 + \alpha_2 \in \Delta_0$, then $k^{\alpha_1} + k^{\alpha_2} = 0$. The latter easily follows by considering the coefficients at $g_{\alpha_1+\alpha_2}$ in $\phi(g_{\alpha_1})g_{\alpha_2} + g_{\alpha_1}\phi(g_{\alpha_2})$ and $\phi(g_{\alpha_1+\alpha_2})$. Note that this coefficient is zero in the case $\phi(g_{\alpha_1+\alpha_2})$. Then

$$\begin{aligned}k^{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)} = -k^{\frac{1}{2}(\epsilon_1-\epsilon_2-\epsilon_3-\delta)} = k^{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3+\delta)} = -k^{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)},\\ k^{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)} = -k^{\frac{1}{2}(\epsilon_1-\epsilon_2-\epsilon_3+\delta)} = k^{\frac{1}{2}(-\epsilon_1-\epsilon_2-\epsilon_3-\delta)} = -k^{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)},\end{aligned}$$

i. e., $k^{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)} = k^{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)} = 0$.

Now, we may deduce the following equalities:

$$\begin{aligned}\phi(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}) = \phi(hg_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}) &= -h\phi(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}) - \phi(h)g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)} = \\ l^{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)} &- (bg_\delta + cg_{-\delta})g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)} \text{ and} \\ \phi(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)}) = -\phi(hg_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)}) &= \phi(h)g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)} + h\phi(g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)}) = \\ (bg_\delta + cg_{-\delta})g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3-\delta)} &+ l^{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)}g_{\frac{1}{2}(\epsilon_1+\epsilon_2+\epsilon_3+\delta)},\end{aligned}$$

whence $b = c = 0$.

Thus, ϕ is trivial on $(F(4))_0$. By Lemma 9 we obtain the required assertion. The lemma is proved.

Lemma 18. The superalgebra $G(3)$ does not admit nonzero antiderivations.

Proof. Assume that ϕ is a nontrivial antiderivation of $G(3)$. It is clear that $(G(3))_0 = A_1 \oplus \mathbf{G}_2$. By Lemmas 4–7, we have $\phi(A_1) \subseteq A_1$, $\phi(\mathbf{G}_2) = 0$, $\phi((G(3))_1) \subseteq (G(3))_1$. Assume that ϕ acts on A_1 by the standard way, i. e.,

$$\phi(h) = -2ah + bg_{2\delta} + cg_{-2\delta},\ \phi(g_{2\delta}) = 2ch + ag_{2\delta} + dg_{-2\delta},\ \phi(g_{-2\delta}) = 2bh + eg_{2\delta} + ag_{-2\delta}.$$

Let

$$\phi(g_{\epsilon_i+(-1)^l\delta}) = k_{i,l}^{\delta}g_\delta + k_{i,l}^{-\delta}g_{-\delta} + \sum_{j=1}^{3} k_{i,l}^{\pm\epsilon_j\pm\delta}g_{\pm\epsilon_j\pm\delta}, \quad l = 1, 2.$$

If $g_{\epsilon_i+(-1)^l\delta}g_{-\epsilon_i} = \beta_{i,l}g_{(-1)^l\delta}$, $\beta_{i,l} \in F$, $l = 1, 2$, then

$$\begin{aligned}\beta_{i,l}\phi(g_{(-1)^l\delta}) &= \phi(g_{\epsilon_i+(-1)^l\delta}g_{-\epsilon_i}) = -(k_{i,l}^{\delta}g_\delta + k_{i,l}^{-\delta}g_{-\delta} + \sum_{j=1}^{3} k_{i,l}^{\pm\epsilon_j\pm\delta}g_{\pm\epsilon_j\pm\delta})g_{-\epsilon_i} = \\ -(k_{i,l}^{\delta}g_\delta g_{-\epsilon_i} &+ k_{i,l}^{-\delta}g_{-\delta}g_{-\epsilon_i} + k_{i,l}^{\epsilon_i\pm\delta}g_{\epsilon_i\pm\delta}g_{-\epsilon_i} + \sum_{j=1,j\neq i}^{3} k_{i,l}^{-\epsilon_j\pm\delta}g_{-\epsilon_j\pm\delta}g_{-\epsilon_i}).\end{aligned}$$

By the arbitrariness of i and $\epsilon_1+\epsilon_2+\epsilon_3=0$, we obtain $\phi(g_{\pm\delta})\in \mathrm{span}\langle g_\delta, g_{-\delta}\rangle$.

Consider the subsuperalgebra $B=A_1\oplus sl_2$ in $G(3)$; B is a basic classical Lie superalgebra of type $B(0,1)$. As it was shown above, B is invariant under ϕ. Thus, Lemma 13 implies the triviality of ϕ on B, and, in particular, on A_1, which implies the triviality ϕ on $(G(3))_0$. By Lemma 9 we obtain the required assertion. The lemma is proved.

It remains to consider the case of non-basic classical Lie superalgebras, that do not satisfy the conditions of Theorem 2. Therefore, we consider these superalgebras from a general argument.

Lemma 19. Let ϕ be a nontrivial δ-derivation of $A(1,1)$. Then $\delta=\frac{1}{2}$ and $\phi(x)=\alpha x$, $\alpha\in F$.

Proof. Let ϕ be a nontrivial δ-derivation of $A(1,1)$. It suffices to consider three cases $\delta=-1$, $\delta=\frac{1}{2}$, and $\delta\neq -1,0,\frac{1}{2},1$.

Let $\delta=-1$. Assume that $\phi(e_{ij})=\sum\limits_{k,l=1}^{4}\gamma_{kl}^{ij}e_{kl}$ holds for $e_{ij}\in (A(1,1))_1$, and

$$\phi(e_{12})=2c(e_{11}-e_{22})+ae_{12}+de_{21},\quad \phi(e_{21})=2b(e_{11}-e_{22})+ee_{12}+ae_{21},$$

$$\phi(e_{11}-e_{22})=-2a(e_{11}-e_{22})+be_{12}+ce_{21},\ \phi(e_{34})=2c^*(e_{33}-e_{44})+a^*e_{34}+d^*e_{43},$$
$$\phi(e_{43})=2b^*(e_{33}-e_{44})+e^*e_{34}+a^*e_{43},\ \phi(e_{33}-e_{44})=-2a^*(e_{33}-e_{44})+b^*e_{34}+c^*e_{43},$$

for the even elements. Also we have

$$\phi(e_{13})=\phi((e_{11}-e_{22})e_{13}) \ = \ -((-2a(e_{11}-e_{22})+be_{12}+ce_{21})e_{13}+(e_{11}-e_{22})\sum_{k,l=1}^{4}e_{kl}\gamma_{kl}^{13}e_{kl})=$$
$$2ae_{13} \ - \ ce_{23}-(e_{11}-e_{22})\sum_{k,l=1}^{4}e_{kl}\gamma_{kl}^{13}e_{kl}.$$

On the other hand,

$$\phi(e_{13})=\phi(e_{13}(e_{33}-e_{44})) \ = \ -(\sum_{k,l=1}^{4}\gamma_{kl}^{13}e_{kl}(e_{33}-e_{44})+e_{13}(-2a^*(e_{33}-e_{44})+b^*e_{34}+c^*e_{43}))=$$
$$(e_{33} \ - \ e_{44})\sum_{k,l=1}^{4}\gamma_{kl}^{13}e_{kl}+2a^*e_{13}-b^*e_{14},$$

whence $a=a^*=\gamma_{13}^{13}$, $b^*=c=0$, $\phi(e_{13})=ae_{13}+\gamma_{31}^{13}e_{31}+\gamma_{24}^{13}e_{24}$. Therefore,

$$\phi(e_{23})=\phi(e_{21}e_{13}) \ = \ -((2c(e_{11}-e_{22})+ee_{12}+ae_{21})e_{13}+e_{21}(ae_{13}+\gamma_{31}^{13}e_{31}+\gamma_{24}^{13}e_{24}))=$$
$$-2ce_{13} \ - \ 2ae_{23},$$

which gives $\phi(e_{13})=\phi(e_{12}e_{23})=-((ae_{12}+de_{21})e_{23}+e_{12}(-2ce_{13}-2ae_{23}))=ae_{13}$.

By an analogous argument for $\phi(e_{31})$, we get

$$\phi(e_{31})=\phi(e_{31}(e_{11}-e_{22}))=(e_{11}-e_{22})\sum_{k,l=1}^{4}\gamma_{kl}^{31}e_{kl}+2ae_{31}-be_{32},$$

$$\phi(e_{31})=\phi((e_{33}-e_{44})e_{31})=2ae_{31}-c^*e_{41}-(e_{33}-e_{44})\sum_{k,l=1}^{4}\gamma_{kl}^{31}e_{kl}.$$

These relations give $b = c^* = 0$, $\phi(e_{31}) = ae_{31} + \gamma_{13}^{31}e_{13} + \gamma_{42}^{31}e_{42}$, whence $\phi(e_{41}) = \phi(e_{43}e_{31}) = -2ae_{41}$ and $\phi(e_{31}) = \phi(e_{34}e_{41}) = ae_{31}$. Now, $\phi(e_{34}) = \phi(e_{31}e_{14}) = ae_{34}$, and $\phi(e_{43}) = \phi(e_{41}e_{13}) = ae_{43}$, i. e., $d^* = e^* = 0$.

From $\phi(e_{23}) = \phi(e_{23}(e_{33} - e_{44})) = 4ae_{23}$ we infer that $a = 0$ and $\phi(e_{23}) = 0$. Analogously, we deduce $\phi(e_{14}) = \phi(e_{24}) = \phi(e_{42}) = \phi(e_{32}) = 0$.

The equalities $d = e = 0$ follow from $\phi(e_{13}) = 0, \phi(e_{32}) = 0$, and

$$\phi(e_{12}) = -\phi(e_{13})e_{32} - e_{13}\phi(e_{32}) = 0, \quad \phi(e_{21}) = -\phi(e_{23})e_{31} - e_{23}\phi(e_{31}) = 0.$$

Therefore, ϕ is trivial.

Let $\delta = \frac{1}{2}$. Then $\phi(e_{11} - e_{22}) = \alpha(e_{11} - e_{22})$, $\phi(e_{21}) = \alpha e_{21}$, $\phi(e_{21}) = \alpha e_{21}$, $\phi(e_{33} - e_{44}) = \beta(e_{33} - e_{44})$, $\phi(e_{34}) = \beta e_{34}$, $\phi(e_{43}) = \beta e_{43}$. Note that

$$\phi(e_{13}) = \phi((e_{11} - e_{22})e_{13}) = \frac{1}{2}(\alpha(e_{11} - e_{22})e_{13} + (e_{11} - e_{22})\phi(e_{13})),$$

whence $\phi(e_{13}) = \alpha e_{13}$. On the other hand, $\phi(e_{13}) = \phi(e_{13}(e_{33}-e_{44})) = \frac{1}{2}(\alpha+\beta)e_{13}$, which implies $\alpha = \beta$. It is clear that $\phi(e_{23}) = \phi(e_{21}e_{13}) = \alpha e_{23}$, $\phi(e_{14}) = \phi(e_{13}e_{34}) = \alpha e_{14}$, $\phi(e_{24}) = \phi(e_{21}e_{14}) = e_{24}$. Analogously, we deduce $\phi(e_{41}) = \alpha e_{41}$, $\phi(e_{42}) = \alpha e_{42}$, $\phi(e_{31}) = \alpha e_{31}$, $\phi(e_{32}) = \alpha e_{32}$, i. e., $\phi(x) = \alpha x$ for an arbitrary $x \in A(1,1)$.

Let $\delta \neq -1, 0, \frac{1}{2}, 1$. In this case, $\phi(x) = 0$ for $x \in (A(1,1))_0$. Obviously, for $e_{ij} \in (A(1,1))_1$, we have $\phi(e_{ij}) = \pm\phi((e_{11} - e_{22})e_{ij}) = \pm\delta(e_{11} - e_{22})\phi(e_{ij})$, whence $\phi(e_{ij}) = 0$, i. e., ϕ is trivial. The lemma is proved.

Since $(P(n))_0$ and $(Q(n))_0$ do not contain a simple subalgebra of dimension $d \leq 3$ as a direct summand, we may conclude that the nontrivial δ-derivations of $P(n)$ and $Q(n)$ are zero on $(P(n))_0$ and $(Q(n))_0$ when $\delta \neq \frac{1}{2}$.

Lemma 20. Let ϕ be a nontrivial δ-derivation of $Q(n)$. Then $\delta = \frac{1}{2}$ and $\phi(x) = \alpha x, \alpha \in F$.

Proof. Let $t = 2n + 2$, $a_{i,j} = e_{i,j} + e_{n+1+i,n+1+j} + E$, $b_{i,j} = e_{i,n+1+j} + e_{n+1+i,j} + E$, $c_{i,j} = e_{i,n+1+i} + e_{n+1+i,i} - e_{j,n+1+j} - e_{n+1+j,j}$, and $\phi(b_{i,j}) = \sum\limits_{k,l=1}^{t} \lambda_{k,l}^{i,j}e_{k,l} + E$. It is clear that if $\delta \neq \frac{1}{2}$ then $\phi((Q(n))_0) = 0$. From

$$a_{i,i}b_{i,j} = b_{i,j}, \quad b_{i,j}a_{j,j} = b_{i,j} \tag{3}$$

it easily follows that $\phi(b_{i,j}) = \delta a_{i,i}\phi(b_{i,j})$ and $\phi(b_{i,j}) = \delta\phi(b_{i,j})a_{j,j}$, whence

$$\phi(b_{i,j}) = \delta\left(\sum_{l=1}^{t} \lambda_{i,l}^{i,j}e_{i,l} - \sum_{k=1}^{t} \lambda_{k,i}^{i,j}e_{k,i} + \sum_{l=1}^{t} \lambda_{n+1+i,l}^{i,j}e_{n+1+i,l} - \sum_{k=1}^{t} \lambda_{k,n+1+i}^{i,j}e_{k,n+1+i} + E\right),$$

i. e., $\phi(b_{i,j}) = \sum\limits_{k=1}^{t} \lambda_{k,i}^{i,j}e_{k,i} + \sum\limits_{k=1}^{t} \lambda_{k,n+1+i}^{i,j}e_{k,n+1+i} + E$. The latter implies

$$\phi(b_{i,j}) = \lambda_{j,i}^{i,j}e_{j,i} + \lambda_{j,n+1+i}^{i,j}e_{j,n+1+i} + \lambda_{n+1+j,i}^{i,j}e_{n+1+j,i} + \lambda_{n+1+j,n+1+i}^{i,j}e_{n+1+j,n+1+i} + E,$$

which gives $\phi(b_{k,j}) = \phi(a_{k,i}b_{i,j}) = \delta a_{k,i}\phi(b_{i,j}) = 0$. Thus, $\phi(c_{k,i}) = \phi(b_{k,i}a_{i,k}) = 0$, i. e., $\phi = 0$ by the linearity of ϕ.

By Lemma 5, $\phi(x) = \alpha x$, $\alpha \in F$, $x \in (Q(n))_0$ when $\delta = \frac{1}{2}$, whence by (3) we have

$$\begin{aligned} \phi(b_{i,j}) \quad &= \quad \phi(b_{i,j}a_{j,j}) = \\ \frac{1}{2}(\alpha b_{i,j} + \sum_{k=1}^{t} \lambda_{k,j}^{i,j}e_{k,j} - \sum_{l=1}^{t} \lambda_{j,l}^{i,j}e_{j,l} \quad &+ \quad \sum_{k=1}^{t} \lambda_{k,n+1+j}^{i,j}e_{k,n+1+j} - \sum_{l=1}^{t} \lambda_{n+1+j,l}^{i,j}e_{n+1+j,l} + E). \end{aligned}$$

The latter implies $\phi(b_{i,j}) = \alpha b_{i,j}$, whence $\phi(c_{k,i}) = \phi(b_{k,i}a_{i,k}) = \alpha c_{k,i}$. Thus, $\phi(x) = \alpha x$, $x \in Q(n)$. The lemma is proved.

Lemma 21. Let ϕ be a nontrivial δ-derivation of $P(n)$. Then $\delta = \frac{1}{2}$ and $\phi(x) = \alpha x$, $\alpha \in F$.

Proof. Let $t = 2n+2$, $a_{i,j} = e_{i,j} - e_{n+1+j,n+1+i}$, $a^{i,j} = e_{i,i} - e_{j,j} + e_{n+1+j,n+1+j} - e_{n+1+i,n+1+i}$, $b_{i,j} = e_{i,n+1+j} + e_{j,n+1+i}$, $c_{i,j} = e_{n+1+i,j} - e_{n+1+j,i}$, $\phi(b_{i,i}) = \sum\limits_{q,l=1}^{t} \nu_{q,l}^{i,i} e_{q,l}$, $\phi(c_{i,j}) = \sum\limits_{q,l=1}^{t} \lambda_{q,l}^{i,j} e_{q,l}$. By Lemma 5, $\phi(P(n)_0) = 0$ when $\delta \neq \frac{1}{2}$. From $\phi(2b_{i,i}) = \phi(a^{i,k}b_{i,i}) = \delta a^{i,k}\phi(b_{i,i})$ we get

$$\begin{aligned}\phi(b_{i,i}) &= \frac{\delta}{2}(\sum_{l=1}^{t}\nu_{i,l}^{i,i}e_{i,l} - \sum_{q=1}^{t}\nu_{q,i}^{i,i}e_{q,i} - \sum_{l=1}^{t}\nu_{k,l}^{i,i}e_{k,l} + \sum_{q=1}^{t}\nu_{q,k}^{i,i}e_{q,k} + \\ \sum_{l=1}^{t}\nu_{n+1+k,l}^{i,i}e_{n+1+k,l} &- \sum_{q=1}^{t}\nu_{q,n+1+k}^{i,i}e_{q,n+1+k} - \sum_{l=1}^{t}\nu_{n+1+i,l}^{i,i}e_{n+1+i,l} + \sum_{q=1}^{t}\nu_{q,n+1+i}^{i,i}e_{q,n+1+i}),\end{aligned}$$

whence $\phi(b_{i,i}) = 0$. Therefore,

$$\phi(b_{j,i}) = \frac{1}{2}\phi(a_{j,i}b_{i,i}) = \frac{\delta}{2}(\phi(a_{j,i})b_{i,i} + a_{j,i}\phi(b_{i,i})) = 0.$$

It is easy to obtain $\phi(c_{i,j}) = 0$, because of

$$c_{i,j} = c_{i,j}a^{j,k} = a^{i,k}c_{i,j}. \tag{4}$$

Now, we see

$$\begin{aligned}\phi(c_{i,j}) = \phi(c_{i,j}a^{j,k}) &= \delta\sum_{q,l=1}^{t}\lambda_{q,l}^{i,j}e_{q,l}(e_{j,j} - e_{k,k} - e_{n+1+j,n+1+j} + e_{n+1+k,n+1+k}) = \\ \delta\sum_{l=1}^{t}(\lambda_{l,j}^{i,j}e_{l,j} &- \lambda_{j,l}^{i,j}e_{j,l} - \lambda_{l,k}^{i,j}e_{l,k} + \lambda_{k,l}^{i,j}e_{k,l} - \\ \lambda_{l,n+1+j}^{i,j}e_{l,n+1+j} &+ \lambda_{n+1+j,l}^{i,j}e_{n+1+j,l} + \lambda_{l,n+1+k}^{i,j}e_{l,n+1+k} - \lambda_{n+1+k,l}^{i,j}e_{n+1+k,l}).\end{aligned}$$

Since $\delta \neq \frac{1}{2}, 1$, we have $\phi(c_{i,j}) = \lambda_{j,n+1+j}^{i,j}e_{j,n+1+j}$. Thus, $\phi(c_{i,k}) = \phi(c_{i,j}a_{j,k}) = \delta\phi(c_{i,j})a_{j,k} = 0$. Hence, ϕ is trivial.

By Lemma 5, $\phi(a_{i,j}) = \alpha a_{i,j}$ and $\phi(a^{i,j}) = \alpha a^{i,j}$ when $\delta = \frac{1}{2}$. By $\phi(2b_{i,i}) = \phi(a^{i,k}b_{i,i})$, we arrive at

$$\begin{aligned}\phi(b_{i,i}) = \frac{1}{4}(2\alpha b_{i,i} + \sum_{l=1}^{t}\nu_{i,l}^{i,i}e_{i,l} - \sum_{q=1}^{t}\nu_{q,i}^{i,i}e_{q,i} - \sum_{l=1}^{t}\nu_{k,l}^{i,i}e_{k,l} + \sum_{q=1}^{t}\nu_{q,k}^{i,i}e_{q,k} + \\ \sum_{l=1}^{t}\nu_{n+1+k,l}^{i,i}e_{n+1+k,l} - \sum_{q=1}^{t}\nu_{q,n+1+k}^{i,i}e_{q,n+1+k} - \sum_{l=1}^{t}\nu_{n+1+i,l}^{i,i}e_{n+1+i,l} + \sum_{q=1}^{t}\nu_{q,n+1+i}^{i,i}e_{q,n+1+i}),\end{aligned}$$

whence $\phi(b_{i,i}) = \alpha b_{i,i}$. Therefore,

$$\phi(b_{j,i}) = \frac{1}{2}\phi(a_{j,i}b_{i,i}) = \frac{1}{4}(\phi(a_{j,i})b_{i,i} + a_{j,i}\phi(b_{i,i})) = \alpha b_{j,i}.$$

By (4) we analogously obtain $\phi(c_{i,j}) = \alpha c_{i,j}$. Thus, $\phi(x) = \alpha x$, $x \in P(n)$. The lemma is proved.

Theorem 22. Let A be a classical Lie superalgebra, and let ϕ be a nontrivial δ-derivation of A. Then $\delta = \frac{1}{2}$ and $\phi(x) = \alpha x$ for some $\alpha \in F$ and an arbitrary $x \in A$.

Proof. follows from Theorem 1 and Lemmas 9–21.

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